PSCAD Cookbook

Circuit Response Studies

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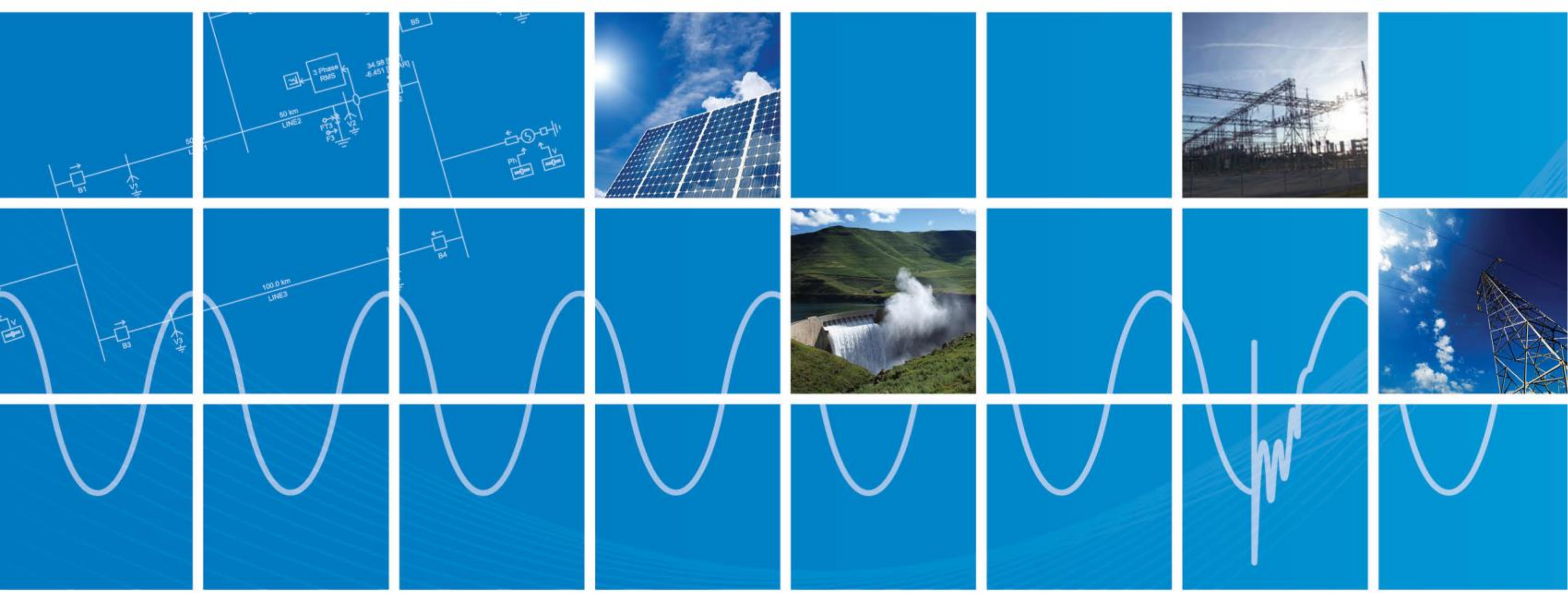

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Contents

3. Circuit Response Studies

3.1 Response Study: Series RL Circuit to a DC Source

Motivation

In this study, a DC source is connected to an RL load through a circuit breaker. While this circuit is very simple, it does have far-reaching implications regarding the behaviour of power transmission and distribution circuits. In fact, many problems in power systems engineering can be broken down into the analysis of simple RL (and C) circuits. For instance, when faults occur in transmission lines, the system behavior is dominated by its RL circuit response. More detailed applications are provided in Sections 3.2 and 3.3.

Analysis

The behavior of a simple RL circuit is governed by the differential equation:

$$V = R \cdot i + L \cdot \frac{di}{dt} \qquad 3\text{-}1$$

The solutions to Equation 3-1 are:

$$I = \frac{V}{R} \cdot \left(1 - e^{-\left(\frac{R}{L}\right) \cdot t} \right) \qquad 3\text{-}2 \qquad\qquad I = \frac{V}{R} \cdot \left(1 - e^{-\left(\frac{t}{\tau}\right) \cdot t} \right) \qquad 3\text{-}3$$

Where τ in Equation 3-3 is the time constant of the response; that is, the 'signature' of the circuit.

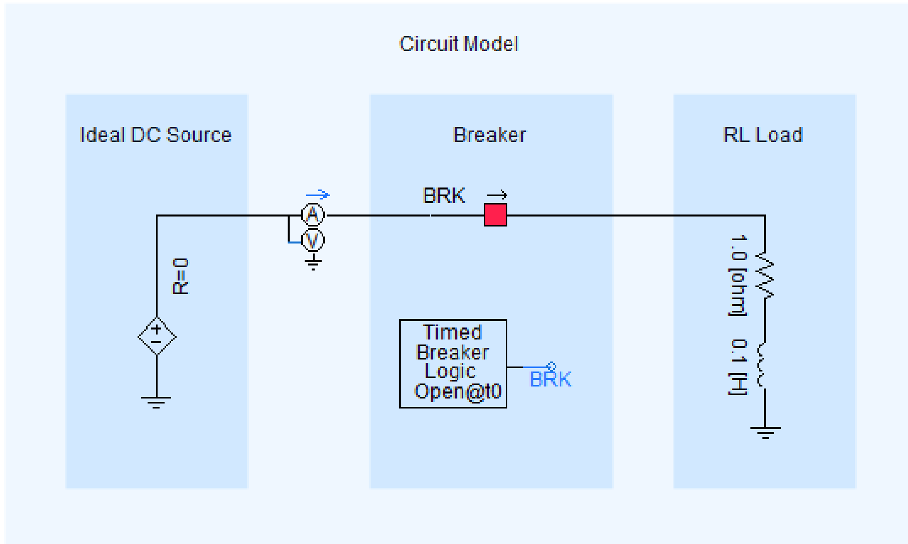

Figure 1: Simple RL Circuit

Consider the circuit shown in Figure 1. For the given component values with a 10 V DC source voltage applied, the equation becomes:

$$I = 10 \cdot \left(1 - e^{-10t}\right) \qquad \text{3-4}$$

In this study, the breaker is initially left open until t = 1.0 s. From basic circuit theory, it is clear that no current can flow while the breaker is open, and thus the output current is zero until t = 1.0 s, and Equation 3-2 becomes:

$$I = 10 \cdot \left(1 - e^{-10(t-1)}\right) \text{ For } t \geq 1 \qquad \text{3-5}$$

Simulation Results

Figure 2 displays the simulation results in PSCAD for the circuit shown in Figure 1; specifically the plotted signals *Vs* and *Is*. These results are consistent with the theoretical results developed above.

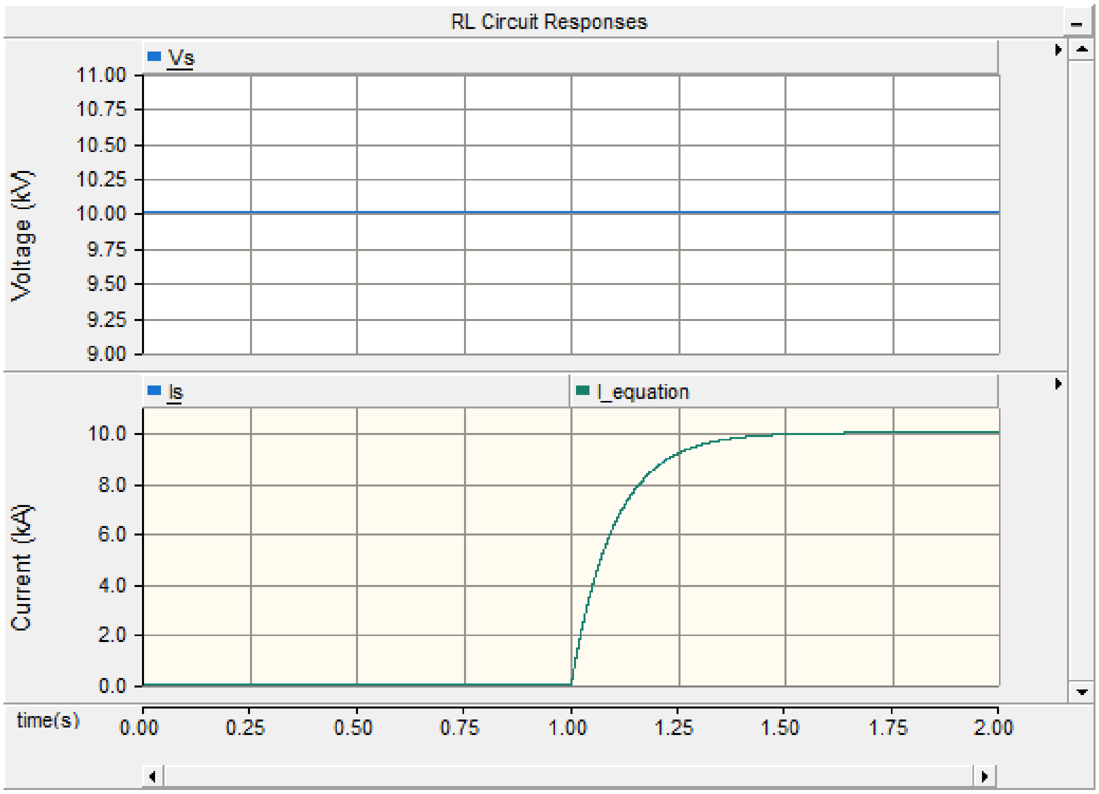

Figure 2: Simulation Results for a Simple RL Circuit

Figure 3 below displays a block diagram of elemental components in PSCAD, constructed to produce a control signal matching Equation 3-5. As can be seen via the plotted signal *I_equation* in Figure 2, the results are consistent with the measured electrical current *Is* (*Is* and *I_equation* are overlaid atop each other).

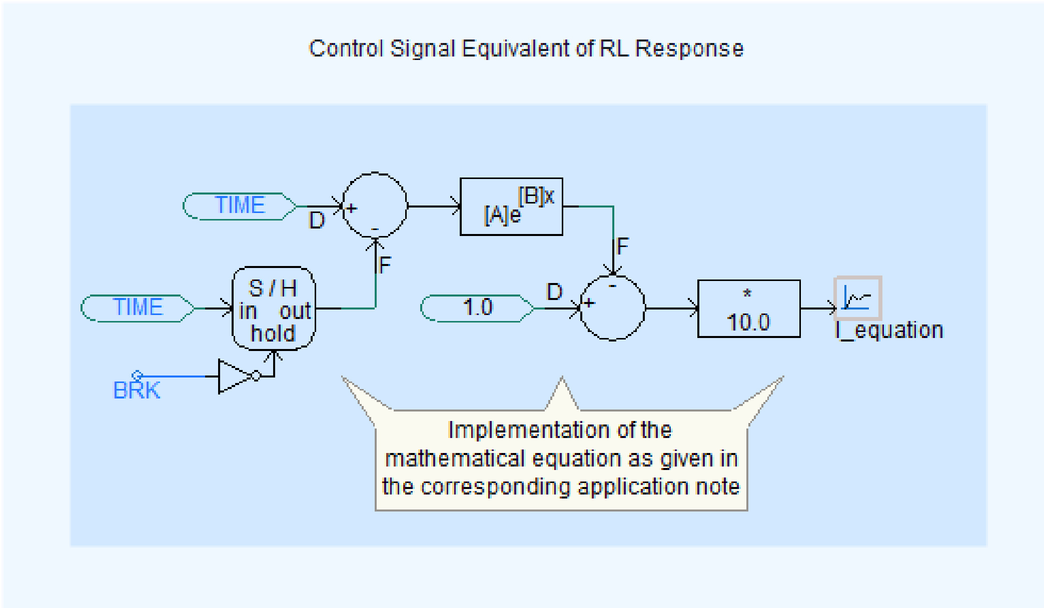

Figure 3: PSCAD Mathematical Component Model Implementation of Equation 3-5

Notes

The 'signature' of this circuit is its time constant, τ, where τ = L/R for an LR circuit.

Discussion

The transient component of the circuit response will decay at a rate determined by the time constant, τ. This idea will be further explored in Sections 3.2 and 3.3, in which an AC source connected to an RL load will be considered.

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Refer to PSCAD case: Circuit_Response_study_01.pscx

3.2 Response Study: Series RL Circuit to an AC (Sinusoidal) Source

Motivation

An AC transmission line is often subject to short circuit faults across its load. When a fault occurs, a transient offset (i.e. DC) current is normally observed. The magnitude of this offset depends on the phase angle of the driving voltage at the time of the fault. This situation can be simplified down to an AC voltage source connected to a series RL circuit. Note that a more detailed study is discussed in Section 3.3.

System Overview

A series RL circuit with a sinusoidal driving source is illustrated in Figure 4. In this particular circuit, the source is a 230 kV, 60 Hz sinusoid and the RL parameters are typical of a transmission line. The resistance of the source is negligible.

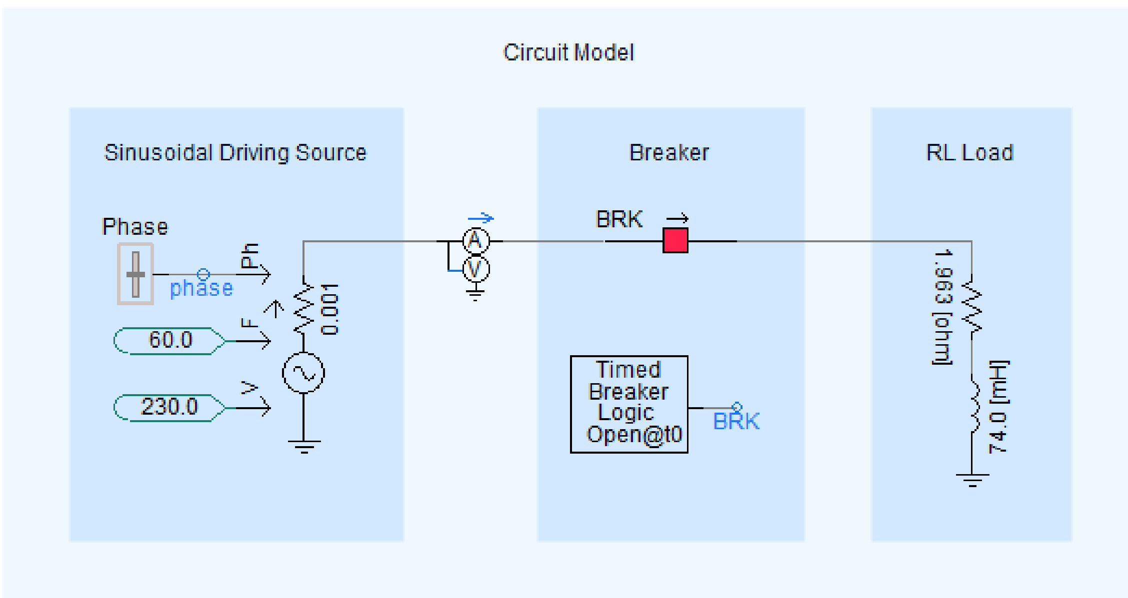


Figure 4: Series RL Circuit

By changing the initial phase of the voltage source, we can affect the observed transient current when switching in the RL circuit. This is referred to as the point on wave impact.

Analysis

The voltage source is defined as:

$$e = E\sqrt{2}\cdot\sin(\omega t+\alpha) \qquad 3\text{-}6$$

In Equation 3-6, α is the initial phase angle.

After the breaker closes, the current in this circuit is given by:

$$I = \frac{E\sqrt{2}}{Z} \cdot \left\{ \sin(\omega t + \alpha - \varphi) - \sin(\alpha - \varphi) \cdot e^{-\frac{t}{\tau}} \right\} \quad 3\text{-}7$$

Where:

$$Z = \sqrt{R^2 + \omega^2 L^2}$$

$$\varphi = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

$$\tau = \frac{L}{R}$$

Note
This derivation implies the breaker closes at t = 0. Angle α represents the point-on-wave position.

The current response is made up of two parts:

- The sinusoidal current;

$$I_{SS} = \frac{E\sqrt{2}}{Z} \cdot \sin(\omega t + \alpha - \varphi) \quad 3\text{-}8$$

- And the exponentially decaying DC offset current;

$$I_{TR} = \frac{E\sqrt{2}}{Z} \cdot \sin(\alpha - \varphi) \cdot e^{-\frac{t}{\tau}} \quad 3\text{-}9$$

The quantities I_{SS} and I_{TR}, as given by Equations 3-8 and 3-9 are shown in Figure 5.

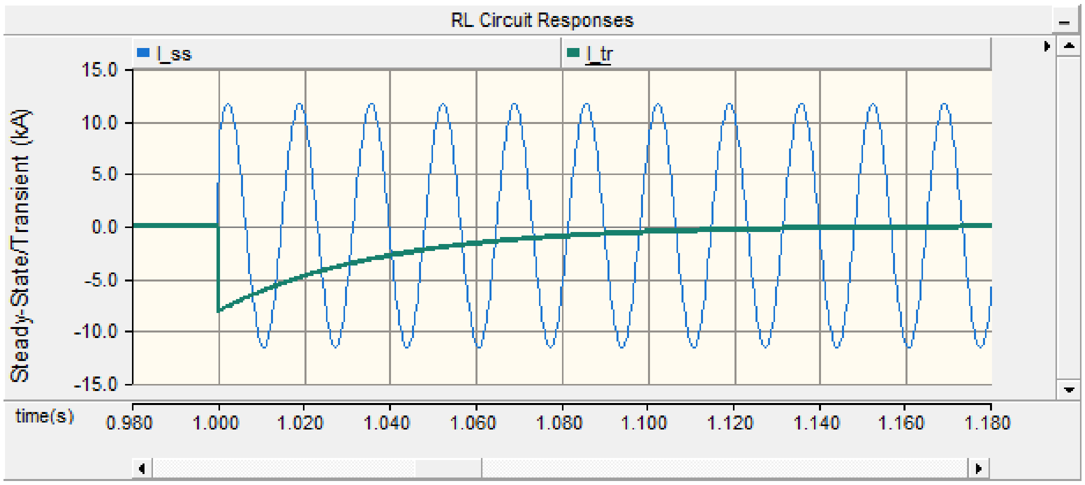


Figure 5: Steady-State and Transient Current Response

If $\alpha = \varphi$ when the breaker is closed, then the offset is zero. If $\alpha - \varphi = \frac{\pi}{2}$ when the breaker is closed, then the offset is maximized.

Simulation Results

The circuit response and mathematical predictions are plotted simultaneously for comparison with various initial phase angles in Figure 6, Figure 7, and Figure 8.

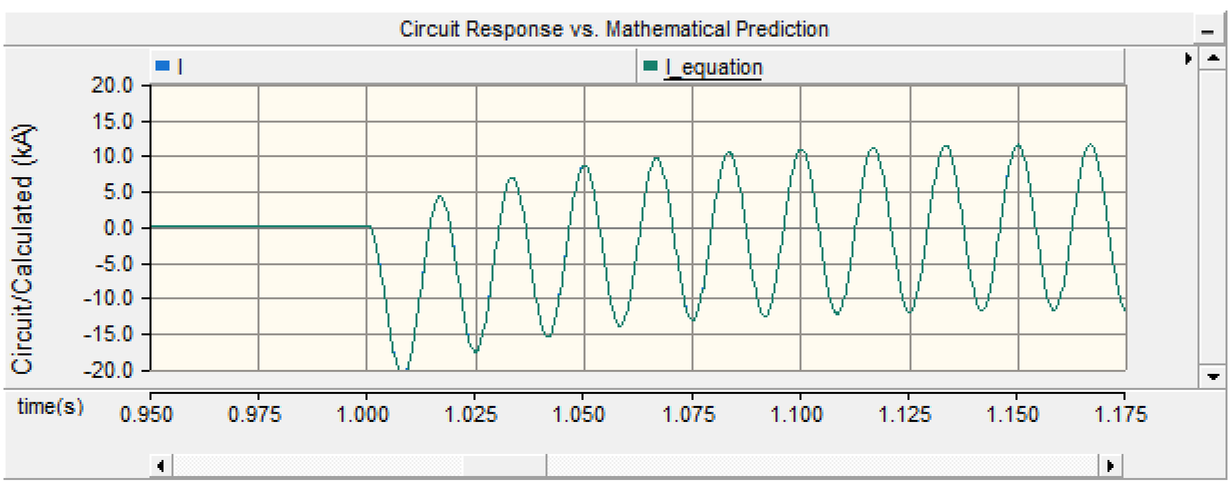


Figure 6: Maximum Transient Offset (here α = 175.98°)

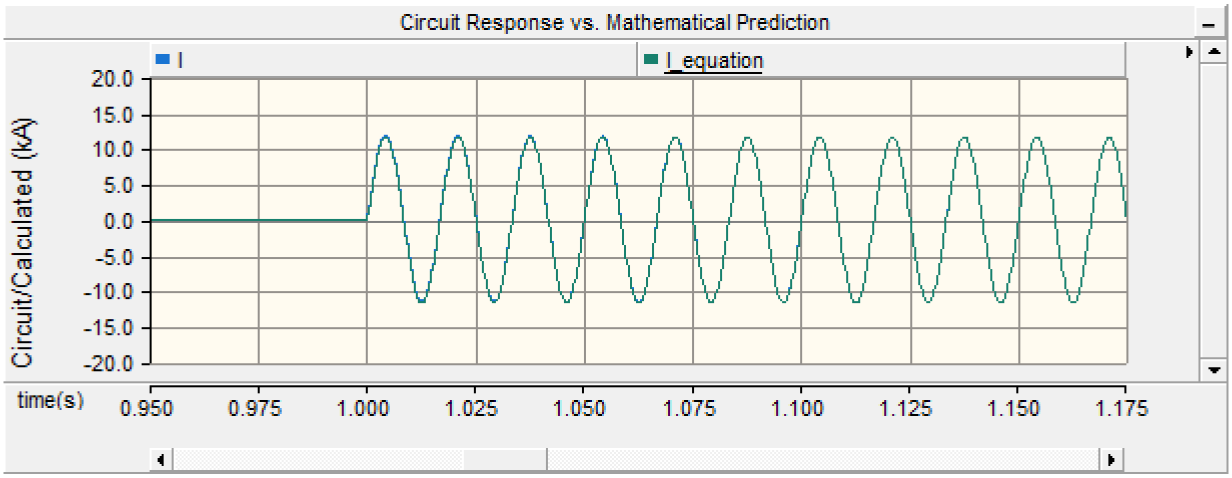


Figure 7: No Transient Offset (here α = 85.98°)

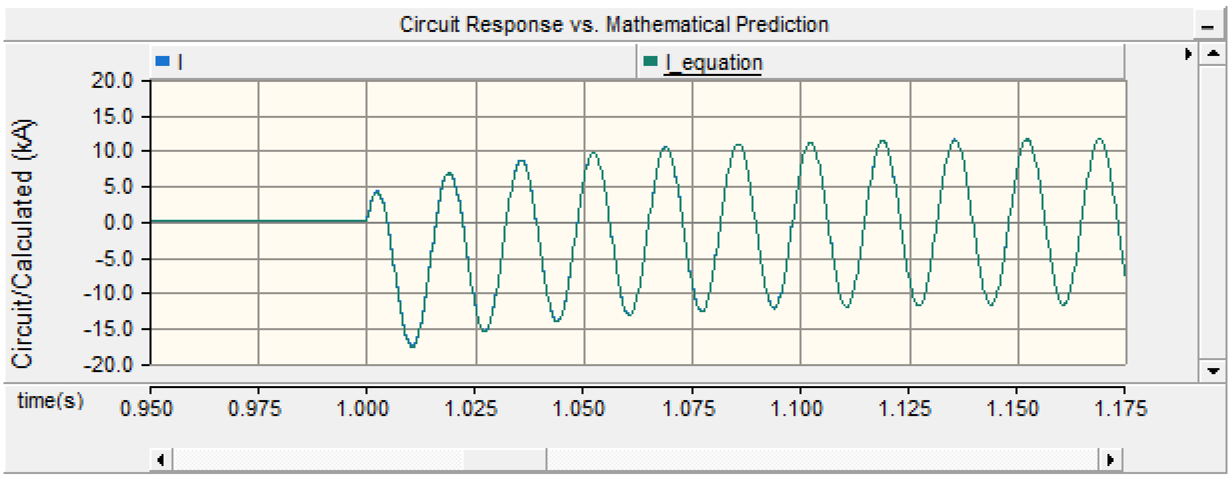


Figure 8: Some Transient Offset (here α = 130.00°)

Discussion

PSCAD results agree with the expected values (equation for current, *I*). As predicted by the mathematical model, there is no transient effect when the source phase angle equals the natural phase of the LR circuit. However, the transient effect is maximized when there is a 90° phase difference between them.

PSCAD

Refer to PSCAD case: Circuit_Response_study_02.pscx

References

1. Prévé, Christophe, "Protection of Electrical Networks", ISTE Ltd.: London, 2006, pp. 82-87.

3.3 Response Study: Series RLC Circuit to an AC (Sinusoidal) Source

Motivation

Understanding the response of a series RLC circuit is very important when analyzing transients in electrical networks. One situation that readily comes to mind is the switching of either capacitors or capacitor banks.

System Overview

In Figure 9, the voltage source and inductor together form the Thévanin equivalent of the power system. The capacitor models a single phase of a three-phase system, and the resistor accounts for losses in the circuit. The contribution of each of the three elements to the nature of the system response is discussed below.

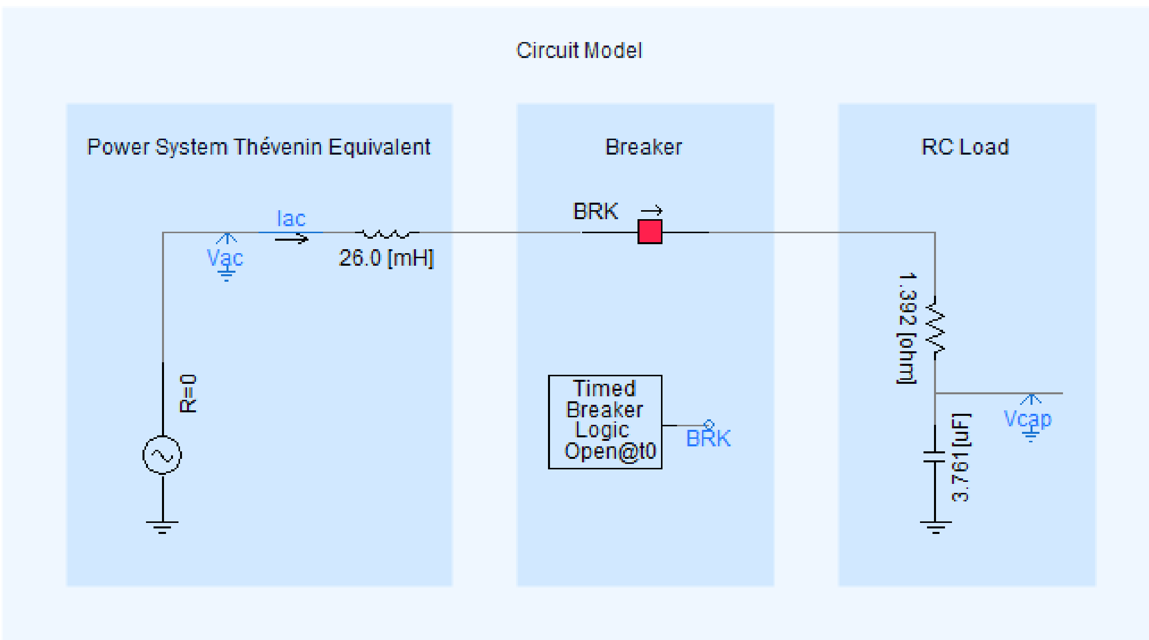


Figure 9: A Simple RLC Circuit

Analysis

From Kirchhoff's Voltage Law, the behavior of the series RLC circuit is given by:

$$V = IR + L\frac{dI}{dt} + \frac{1}{C}\cdot\int i\cdot dt \quad \text{or} \quad \frac{1}{L}\frac{dV}{dt} = \frac{d^2 I}{dt^2} + \frac{R}{L}\frac{dI}{dt} + \frac{1}{LC} \qquad 3\text{-}10$$

And the corresponding characteristic (s-domain) equation is:

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \qquad 3\text{-}11$$

The roots of Equation 3-11 are:

$$s_{1,2} = -\frac{R}{2L} \pm \sqrt{\left(\frac{R}{2L}\right)^2 - \frac{1}{LC}} \quad \text{or} \quad s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0{}^2} \qquad 3\text{-}12$$

Note

$\alpha = \frac{R}{2L}$ is called the damping coefficient and $\omega_o = \frac{1}{\sqrt{LC}}$ is the un-damped resonant frequency.

Depending on the values of the RLC elements, the response can assume three distinct forms:

- Over damped solution:
  - If $R^2 > \frac{4L}{C}$ (i.e.: $\alpha > \omega_o$), then the natural response of the system is given by:

    $$I_n = C_1 e^{-S_1 t} + C_2 e^{-S_2 t}$$

- Critically damped solution:
  - Similarly, if $R^2 = \frac{4L}{C}$ (i.e.: $\alpha = \omega_o$), then the natural response is given by:

    $$I_n = (C_1 + C_2 t)e^{-S_1 t}$$

- Under damped solution:
  - Finally, if $R^2 < \frac{4L}{C}$ (i.e.: $\alpha < \omega_o$), then the natural response is given by:

    $$I_n = e^{-\alpha t}[C_1 Cos\{\sqrt{(\omega_o{}^2 - \alpha^2)}\, t\} + C_2 Sin\{\sqrt{(\omega_o{}^2 - \alpha^2)}\, t\}]$$

For the given component values in Figure 9 and with a source voltage of $132\sqrt{2}Sin(\omega t + \varphi)$ (kV):

$$\omega_o = 3189 Rads/sec \; (f_o = 509Hz) \text{ and } \alpha = 26.77$$

Thus the system is under damped and I_n is given by (in kA):

$$I_n = e^{-26.77 \cdot t}\{C_1 \cdot \cos(2 \cdot \pi \cdot 509 \cdot t) + C_2 \cdot \sin(2 \cdot \pi \cdot 509 \cdot t)\} \quad 3\text{-}13$$

Using phasor analysis, the force response is found to be (this is approximate, since R is small and thus the circuit is assumed to be capacitive):

$$I_f = 0.27 \cdot \cos(\omega t + \varphi) \quad 3\text{-}14$$

Simulation Results

The simulation results shown in Figure 10 yield the same natural frequency, $f_o = 509Hz$, as predicted by the analytical equations (see bottom right of Figure 10).

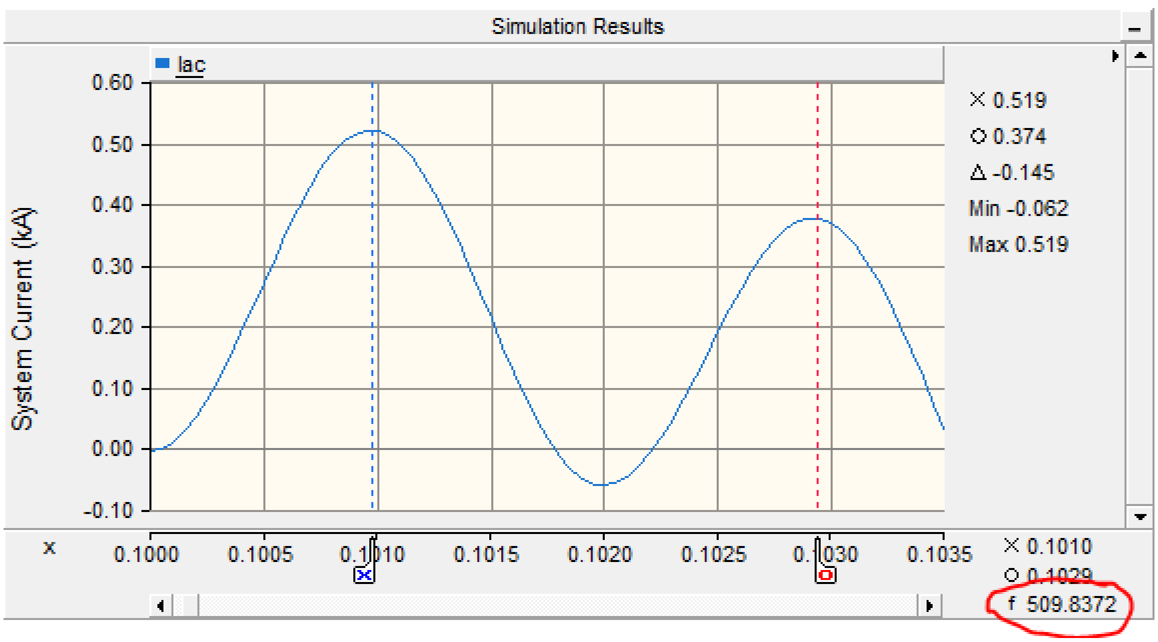


Figure 10: Transient Response of an LRC Circuit

Damping Transients

The equation $\alpha = \frac{R}{2L}$ indicates that changing either R or L will have an effect on the damping of the transient response. Figure 11 shows the response of the circuit with R = 1.392 Ω (Figure 11a) and R = 13.92 Ω (Figure 11b). Clearly, damping increases significantly with an increasing R. Alternatively, increasing L will result in less damping.

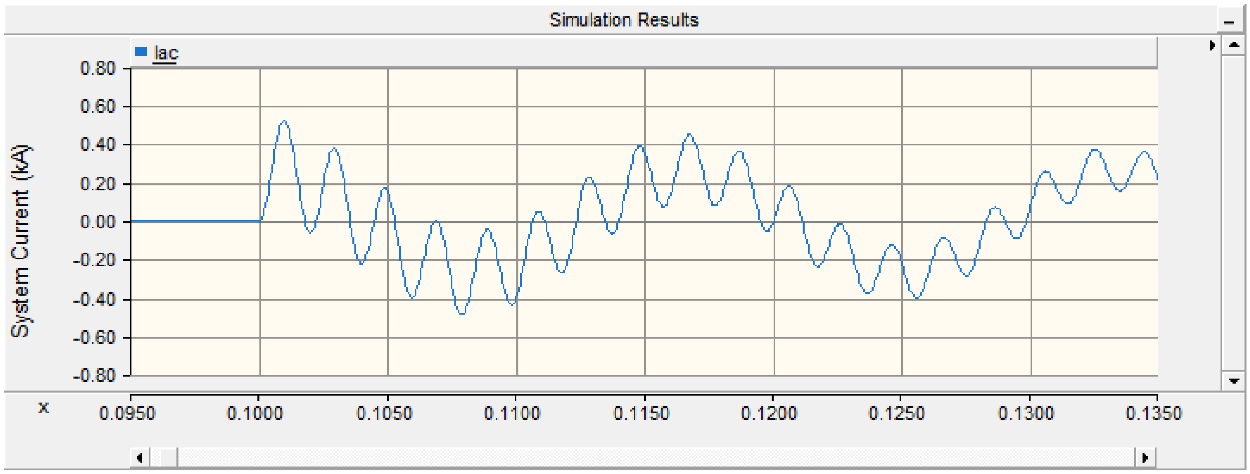


Figure 11a: Effect of Resistance R on Transient Response (1.392 Ω)

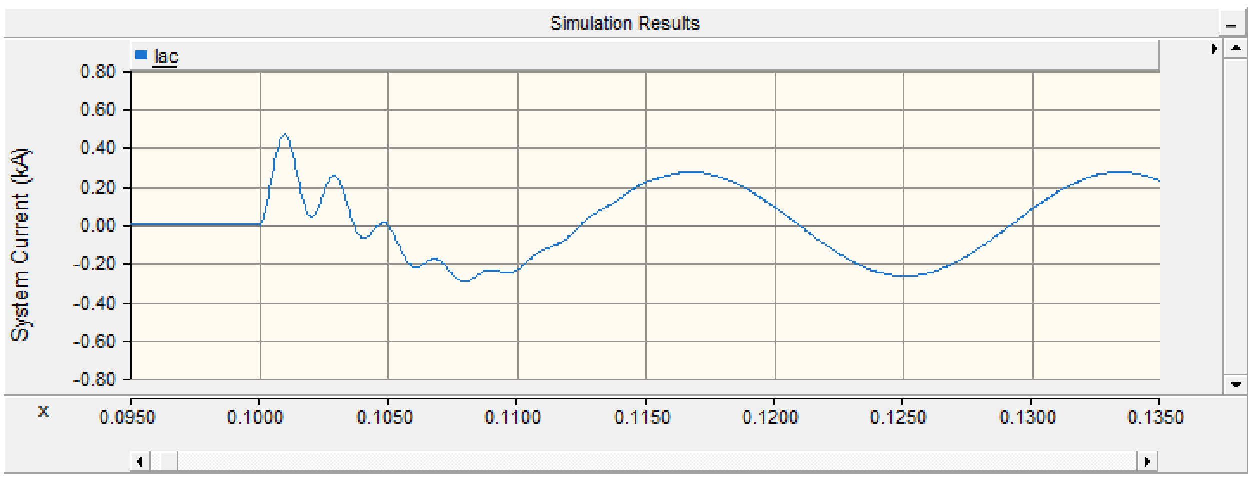
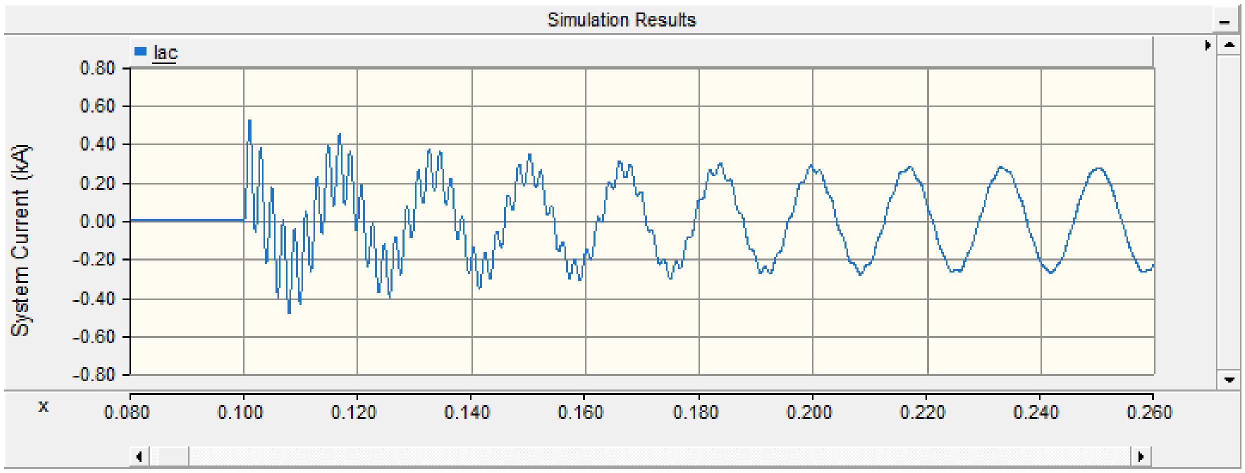


Figure 12b: Effect of Resistance R on Transient Response (13.92 Ω)

As illustrated in Figure 13, once the transient part of the response damps out, the response follows the form of the input to the system. For our circuit, the input is a sinusoid, and as expected the steady-state response is sinusoidal. If the circuit was supplied with a DC source, the steady-state value would be a constant value (in this case, it dies down to zero when the capacitor fully charges).



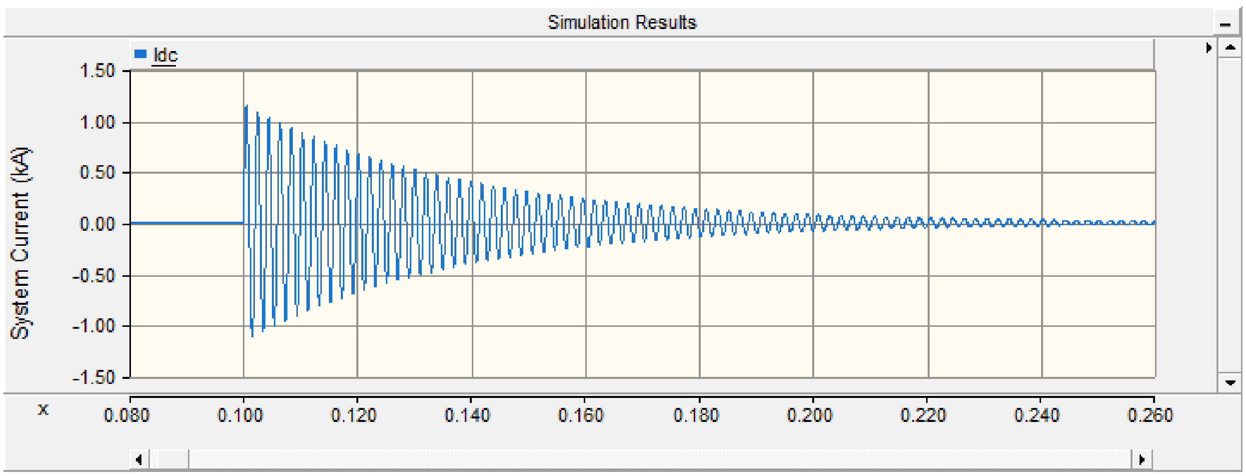


Figure 13: Comparison between Sinusoidal (Top) and DC Source Responses (Bottom)

Discussion

The circuit under consideration exhibited under-damped behavior. This is common in capacitor switching situations due to the fact that capacitor values are typically very small in comparison to R and L. Thus the $R^2 < \frac{4L}{C}$ requirement for under-damped behavior is usually satisfied.

Important Considerations Regarding the PSCAD Simulation Time Step

One important consideration when running PSCAD simulations is that the solution time step, T_s, must be chosen such that it is much less than the time period, T, corresponding to the highest frequency component of currents and voltages under consideration. If this condition is not satisfied, then the output waveform will not be accurate. With the given circuit, the highest frequency component is the natural frequency $f = 509\ Hz$ and $T = 1/f = 1.965\ ms$. Thus it is required that $T_s \ll 1.965\ ms$. As a rule of thumb, T_s should be at least 10 times smaller than T; thus 200 µs is likely a good choice (however, in this simulation, $T_s = 50\mu s$ is used).

Referring to Figure 14, one can easily see the importance of choosing an appropriate solution step size.

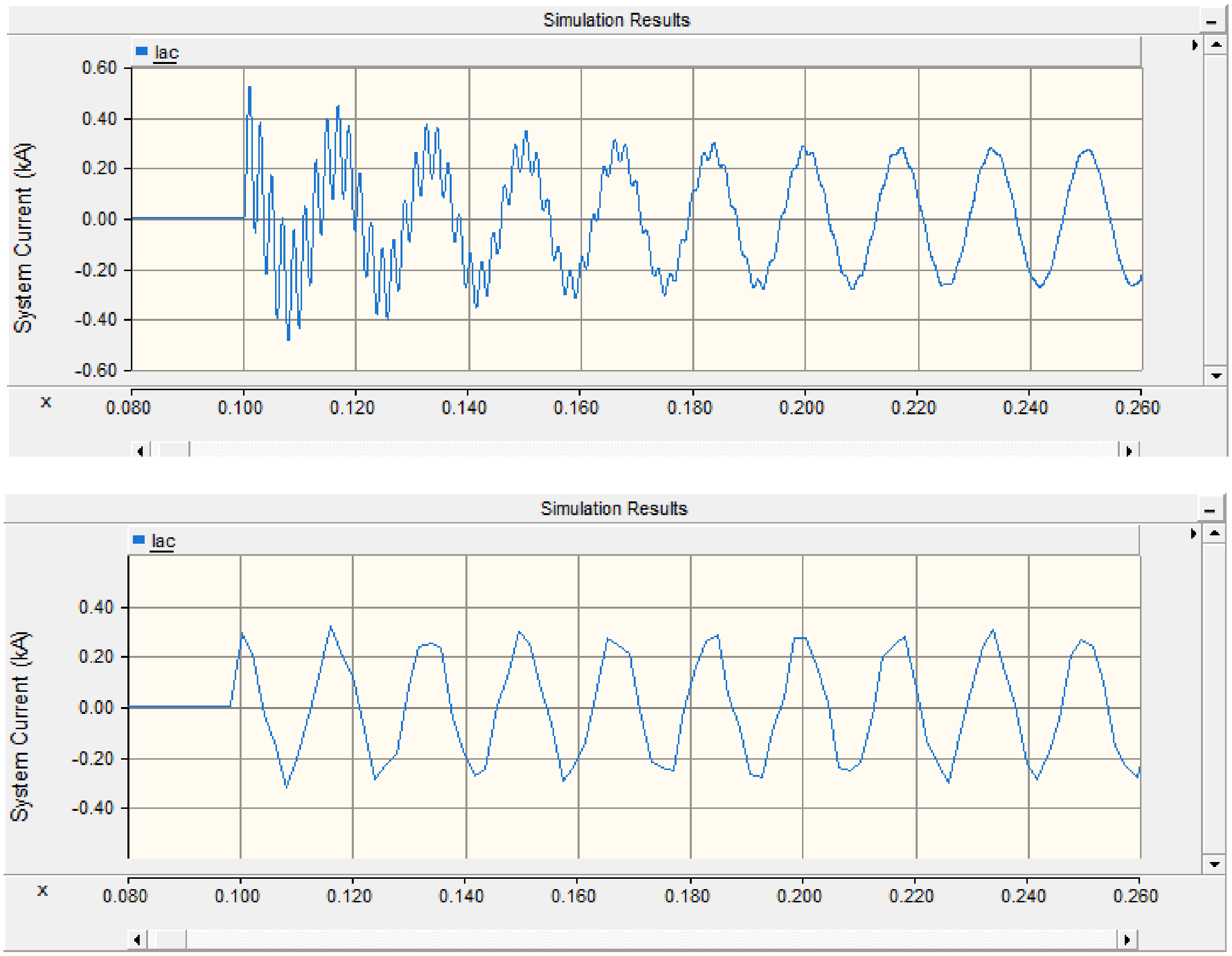


Figure 14 PSCAD Simulation Result with Ts=50µs (Top) and Ts =1.965ms (Bottom)

Note
If only the PSCAD output plot-step is selected too high (time step is okay), the actual calculation will be correct, but the results displayed may lose resolution.

PSCAD

Refer to PSCAD case: Circuit_Response_study_03.pscx

DOCUMENT TRACKING

Rev.	Description	Date
0	Initial	01/Jun/2013
1	Update to new Brand Guidelines	8/May/2018
2	Minor corrections	15/Aug/2018